Money and Walras' Law in the General Theory of Market Disequilibrium

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1. Introduction

Robert A. Gower's insightful paper, "The Keynesian Counter-revolution: A Theoretical Appraisal," and Axel Leijonhuvud's iconoclastic work, On Keynesian Economics and the Economics of Keynes, have led to a renewed interest in the theoretical foundations of macroeconomic analysis. As a result a general theory of disequilibrium behavior has been developed as exemplified by the works of Barro and Grossman, Grossman, and Tucker. That the disequilibrium approach to economic analysis is a most promising innovation is beyond question. At the same time, however, a disquieting consensus with regard to the nature and relevance of the demand for money in the disequilibrium model appears to be emerging. Tucker and Grossman have argued that the demand for money is arbitrary and ambiguous within this context. On this point Grossman has stated:1

...the structure of a monetary economy implies that the excess demand for money, money being the medium of exchange, can have no direct influence upon the formation of prices.

Consequently, we can define the effective excess demand for money arbitrarily.

This line of reasoning is further supported by Tucker:

...in the presence of significant and shifting market disequilibria the demand for money is an ambiguous and arbitrary (and hence tenuous) concept, and this severely limits its usefulness for studying short-run phenomena. (Tucker, 1971, p. 57)

The inability to handle the demand for money within the context of market disequilibrium is symptomatic of a fundamental misunderstanding of the formal structure of the disequilibrium model. In particular, what is at issue is the relevance of Walras' Law. That this is the case is made explicit by Tucker:2

Walras' law is no more than a matter of definition: it is either true or false by definition.

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1 Tucker asserts that the basis for the alleged ambiguity of money demand in his analysis is related to his interpretation of the meaning of ex ante demand functions. The notion of ex ante and ex post relationships can be ambiguous in itself. This is particularly true when used outside the context of the savings-investment relationship. This particular aspect of Tucker's analysis is not considered in this paper except to observe that he defines "the individual's (ex ante) demand for money to be the money balance he desires to be holding at the end of the trading day." (Tucker, 1971, p. 64).

2 If we define the individual's other ex ante demands similarly (i.e., the individual's ex ante demand for consumer goods is the quantity of consumer goods that he desires to have purchased by the end of the trading day) the collection of money demand that Tucker has made in this regard also applies to the other demands in his model. For a further discussion of the general problem of ex ante and ex post relationships see Blackford.
depending on which measure of the demand for money one takes. (Tucker, 1971, p. 69)

Throughout the discussion Tucker makes it clear that Walras' law is taken to be a significant factor in determining the appropriate "measure" of money demand. For example,

The money demand function can be useful ... as a means of obtaining the existing demand for one of the other commodities through Walras' law (and this requires that one use the effective measure of money demand, for which Walras' law is valid). (Tucker, 1971, p. 69)

It will be shown below that these two statements are inaccurate, but for the moment, let us turn to Grossman's formulation of the problem.

In his discussion of the demand for money, Grossman attempts to formulate the problem in terms of the "structure of a market economy in which money serves as a medium of exchange" (Grossman, p. 944). That this is not the basic issue, however, is indicated by his explicit reference to Leijonhufvud: "in this type of model, as Leijonhufvud (1968, p. 88), has pointed out, the specification of the effective excess demand for money is ambiguous." (Grossman, p. 951). If we examine this reference to Leijonhufvud, we find that he is discussing the relevance of Walras' law to the theory of unemployment disequilibrium.

The notion that the demand for money is ambiguous or that it can be arbitrarily defined in such a way as to make Walras' law hold is quite disturbing. After all, if the concept of money demand is ambiguous or arbitrary, it would appear that all of the monetary theory is equally ambiguous or arbitrary. This is a most unsatisfactory state of affairs. It is the purpose of this paper to show that the demand for money is well defined in the general theory of market disequilibrium and to clarify the role of money and money demand within this context. In the following section a choice-theoretic derivation of the familiar Keynesian model of unemployment disequilibrium is briefly outlined.

This derivation is based upon a straightforward application of Clower's dual decision hypothesis. In Section III the relationship between the demand for money and Walras' law is examined within the context of the Keynesian model of unemployment disequilibrium specified in Section II. It is demonstrated that the demand for money is not ambiguous and cannot be arbitrarily defined in such a way as to make Walras' law hold. In Section IV an analytic justification for the assumption that the excess demand for money cannot directly affect prices is given. It is demonstrated that the structure of constraints in the disequilibrium model implies the excess demand for money is always equal to zero.

II. A Choice-Theoretic Keynesian Model

In his discussion of the Keynesian model Clower argued that the actual choice of an individual decision-making unit to buy or sell a particular quantity of a given market is constrained by the units' ability to carry out its planned transactions in other markets, as well as by the vector of prices and state of technology with which the unit is confronted. Specifically, he argued that the quantity of consumer goods a household attempts to purchase in the market is determined not only by the price which it must pay for these goods and the wage at which it can sell its labor services, but also by the quantity of labor services the household is able to sell. Presumably a labor market constraint on the quantity of labor that can be sold will have an effect upon the quantity of consumer goods the household will be willing and able to purchase. Once this presumption is recognized, it is necessary to assume that the choices of an individual in one market are determined by the individual's ability to carry out his desired transactions in other markets. This assumption is the essence of Clower's dual decision hypothesis.

If this hypothesis is to provide a choice-theoretic basis for the familiar Keynesian model, however, Clower's model must be extended to include a monetary sector and to provide explicitly for saving and investment behavior. This extension can be accomplished by introducing real balances and real bonds into the utility function in the manner suggested by Patinkin.

For the purpose of this paper, the Keynesian model with rigid wages and an excess supply of labor is specified within a two sector economy (households and firms) with four goods: real-balances (M), real-bonds (B), labor (L), and output (Y) which outputs consists of both consumption (C) and investment (I) goods. In this economy the general budget constraint faced by the representative household may be written as

\[ w \cdot M + B + w \cdot q = C + I \]

where \( w \) is the real wage, \( B \) and \( M \) are initial holdings of real bonds and real balances, \( q \) is the rate of interest, \( q \) is real profit income, and \( b \) is real interest income. Since there exists an excess supply of labor in the Keynesian model, the representative household's effective demands are derived by maximizing utility subject to (1) and

\[ L = L \]

where \( L \) is the quantity of labor the household can sell and is less than the quantity offered for sale. The behavioral equations implied by this maximization may be written as

\[ C = C(Y, r, B, M, L) \]

\[ B = B(Y, r, B, M) \]

\[ M = M(Y, B, M) \]

where \( Y = w \cdot L + b + q \).

Following Clower (pp. 288-89) and Grossman (p. 951) the household's effective supply of labor is obtained by maximizing utility subject to (1) alone, to obtain

\[ L = L(Y, r, B, M, q, b) \]

Equations (3)-(5) can be referred to as effective demands and (7) as an effective supply in the sense that these equations predict a consistent set of choices made by the household as it is recorded in the market when the quantity of labor that can be sold is restricted; \( C \) represents the quantity of consumer goods the household chooses to buy and is equal to actual purchase, because we are only considering a market constraint on the quantity of labor that can be sold and not on the quantity of consumer goods that can be purchased; \( B \) and \( M \) represent the real quantities of bonds and money that the household chooses to hold and are equal to end of period holdings, and \( L \) represents the quantity of labor services that the household offers for sale and is greater than the quantity actually sold because of the labor market constraint.

The reproductive firm's supply of output, demand for labor, and demand for investment goods can be derived by maximizing the present value of expected future income subject to the production function and various restrictions on costs of adjustment. The behavioral equations implied by this maximization may be written as

\[ \text{Equations (3)-(5)} \]
There is, of course, one more equation than there are unknowns to be solved for. This problem can be eliminated, however, in a manner analogous to that derivation of Walras’s law in the general equilibrium model. Since voluntary exchange implies \( L = L_d \) in the aggregate, substituting from (2)-(5) into (1), together with (11) and rearranging yields

\[
(2 + 4d^2 - 4d - a - r^2) = 0. 
\]

By noting that \( Y^* = W^d d b = q \), (11) can be reduced to

\[
B^* + r^2 d = B^* \quad (\text{or} \ W, B_0). 
\]

which, when used in conjunction with the exogenous demand for money held, can be used to solve for unknowns: the rate of interest, price of output, and the level of output and employment. These five equations denote the equilibrium conditions for the Keynesian model: (13) and (14) denote the familiar IS and LM schedule; (15) and (16) denote the bond and commodity market equilibrium conditions; and (17) denotes that the level of employment is given by the demand for labor in accordance with the assumption of voluntary exchange.

The model is less than zero, and Walras’ law (which requires the sum of the excess demands to be equal to zero) cannot hold. Tucke (1971, p. 69) has suggested that Walras’ law may be rescued by redefining what is meant by the “effective” demand for money. If the individual’s effective demand for money is defined by substituting his other effective demands into the general budget constraint, when constraints are summed across decision making units, Tucker asserts that the effective excess demands of the system sum to zero, as is required by Walras’ law. At this point, however, we are seemingly left with two concepts of money demand: the textbook demand which is implied by the choices of decision making units and for which Walras’ law does not hold, and the effective demand which is implied by the budget constraint and for which Walras’ law does hold. As a result, both Tucker and Grossman have concluded that the entire concept of money demand is ambiguous and arbitrary. If one takes a closer look at the Tucker-Grossman concept of the effective demand for money, however, it can be demonstrated that this conclusion is unwarranted.

The Tucker-Grossman effective demand for money (\( M^e \)) in the Keynesian model is defined as

\[
M^e = \frac{M}{Y} \quad \text{(18)}
\]

This proposition was first demonstrated by Clower, 1977, p. 190. There is a certain irony in Grossman’s defining the effective demand for money as being given “by the budget constraint.” In giving this definition of money demand, he violates his own rule. The model assumes that the individual always demands that he in silver terms, the total amount of money required in the economy, which is equal to the total amount of transactions in the economy. This total quantity may exceed the perceived cost of transactions in the economy, or the concept of money demand is not satisfied by the budget constraint.

4The existence and derivation of this constraint on the Keynesian model are not well known. Tucker (1972, p. 190) has suggested that Patinkin’s derivation of money demand and Walras’ law can be made more precise by adding a constraint on the demand for money via equation (16). This suggestion neglects the point. When the assumptions in Patinkin’s model are properly specified, equation (16) is automatically implied. The specified demand for money is in Patinkin’s model is implied by the same optimization paradigm that underlies the derivation of all the demands and supplies of his model.
M* = M + (B - B')y + W* - L + q - c - b

As defined by (19), M* represents the quantity of money the household will choose to hold if all of its other effective demands are satisfied.

First, contrary to Tucker's (1971, pp. 78-79) assertion, the effective demand for money so defined cannot be treated empirically. The reason for this is that households can never be observed choosing to hold a quantity of money equal to M* if the household is confronted with a labor market constraint on the quantity of labor that it can sell. This would require by way of (19) that all of the household's effective demands be satisfied. But the derivation of c^2 and b^2 presupposes that L^2 is not realized in that L^2 is not the quantity of labor sold but rather the quantity offered for sale. As a result all three of these relationships cannot be satisfied without violating the assumption that the labor market is in a state of excess supply, and households cannot actually choose to hold a quantity of money equal to M*.

Second, accepting M* as the effective demand for money implies that Walras' law holds in the Keynesian model only in a tautological sense. In the economics outlined above the aggregate constraint on the firm sector is given by (11), Alding (19) and (11) and rearranging yields

\[ u^2 + c^2 - y^2 = (M^* - M)^2 - L^2 \]

This equation appears to demonstrate that the sum of the excess demands in the system is equal to zero. However, it is important to recognize that (20) is not simply a statement about the actual excess demand as measured in the market place, but rather a statement about market excess demands and a demand for money which is defined in such a way as to make (20) hold. As a result the statement is circular and empty, as are all statements that are true by definition.

Third, M* does not resolve the extra equation problem in the Keynesian model. Equation (20) can be used by substitution to eliminate either the excess demand for output or bonds from the model. However, in doing this we introduce the excess effective demand for money (M* - M) and the excess supply of labor (L^2 - L^2), neither of which is formally required to solve the system. In addition, we are still left with the other market clearing equation and the excess textbook demand for money (M* - M). The theory of choice underlying the Keynesian model implies a demand for money that has the same legitimacy, as any other defined in the model. Simply defining M* does not allow us to ignore M*.

We are still left with one more equation than there are unknowns. The redundant equation can only be eliminated via (18).

Defining the effective demand for money by substituting the other effective demands and supplies into the budget constraint does not lead to an empirically meaningful behavioral relationship, does not imply Walras' law in a meaningful sense, and does not solve the extra equation problem; M* is a kind of appendix, an ad hoc construct that serves no useful purpose. When viewed from this perspective, the assertion that the concept of money demand is ambiguous or arbitrary is clearly unfounded.

Only the Keynesian case of unemployment disequilibrium has been considered, but extension of the arguments to the other cases is straightforward.

Up to this point only the static structure of the disequilibrium model has been discussed. Having established the money demand-Walras' law ambiguity within this context, we are now in a position to consider the role of money in dynamic disequilibrium analysis.

IV. Money Excess Demand and Dynamic Disequilibrium Analysis

Both Tucker (1972) and Grossman have excluded the excess demand for money from their dynamic analysis of disequilibrium behavior. The legitimacy of this exclusion is not at all self evident, as the liquidity preference-insurable funds debate bears witness. And yet, no analytic justification for excluding the excess demand for money is given (presumably because of the confusion over the demand for money), and no such justification can be found elsewhere in the literature. However, if one examines the structure of constraints in the disequilibrium model, an analytic justification can be found, and it is possible to explain why the excess demand for money cannot directly influence prices. The reason is the structure of constraints in disequilibrium model implies that the excess demand for money is always equal to zero.

In the Keynesian case of labor market disequilibrium the aggregate constraint on the system, equation (18), states that excess demand for money, bonds, and output sum to zero. The derivation of this constraint, however, presupposes that the excess demands for bonds and output are each equal to zero, since it is assumed that each of these markets clears. Therefore, the aggregate constraint on the system implies that the excess demand for money is also equal to zero.

If either the excess demand for output or bonds is not equal to zero we no longer have the Keynesian case. The behavioral equations specified above can no longer be assumed to predict economic behavior, because the optimization problem from which these equations are derived ignores at least one binding constraint on the choices of decision making units. The structure of the model is changed, and the behavioral equations and equilibrium conditions must be respecified. How does this structural change affect the excess demand for money if, in addition to an excess supply of labor, the rate of interest fails to adjust to clear the bond market causing an excess demand for bonds? To answer this question we need only consider the aggregate constraint on the system for this particular case of market disequilibrium.

If only the goods market clears, and in addition to an excess supply of labor there exists an excess demand for bonds, maximizing utility subject to (1) and (2) no longer corresponds to the problem of choice faced by households. At the existing wage rate and rate of interest households are not only constrained by the quantity of labor that can be sold but also by the quantity of bonds that can be purchased. The representative household is forced to maximize utility subject to (1), (2), and (21):

\[ B = B. \]
where $\bar{B}$ is the quantity of bonds the household is able to purchase and is less than the quantity the household offers to purchase. The demand for money and consumer goods implied by this maximization can be written as

$$M_{d}=\frac{1}{1+(\bar{B})^2} \left( Y_r + \bar{B} R_0 + M_0 \right) \tag{22}$$

$$c_{d} = \frac{1}{1+(\bar{B})^2} \left( Y_r + \bar{B} R_0 + M_0 \right) \tag{23}$$

where $\bar{B}$ becomes an independent variable in these two behavioral relations. Since voluntary exchange implies $I = L - \bar{B}$ and $\bar{B} = \bar{B}^d$ in the aggregate, the aggregate constraint on the household sector can be derived by substituting (2) and (21) into (1) to obtain:

$$c_{d} + (\bar{B} - B_0)\bar{r} + (M_{d} - M_0) = 0 \tag{24}$$

This is the long side of the bond market and is unaffected by the excess demand for bonds. As a result the aggregate constraint on the firm sector is still given by (11). To obtain the aggregate constraint on the system as a whole (11) is subtracted from (24) to obtain:

$$c_{d} + (\bar{B} - B_0)\bar{r} + (M_{d} - M_0) = 0 \tag{25}$$

This constraint states that the excess demand for output and money sum to zero. Its derivation presupposes that the excess demand for output is equal to zero, since it is assumed that the goods market clears. Therefore when there exists an excess supply of labor and an excess demand for bonds, the aggregate constraint on the system implies the excess demand for money is equal to zero as in the Keynesian case.

In the three-sector economy under consideration we have only considered two of the twenty-six conceptually possible cases of market disequilibrium. There is little to be gained, however, by going through a formal derivation of the aggregate constraint on the system for the remaining twenty-four cases. The algebra of the situation is such, that whenever a market fails to clear, the quantity transacted in that market is given (in accordance with the assumption of voluntary exchange) by the short side of the market. This quantity appears in both the firm and household sector's constraints. When these two constraints are aggregated the quantities transacted in disequilibrium markets cancel. The aggregate constraint on the system states that the sum of the excess demands in those markets which clear plus the excess demand for money in equal to zero. Since the excess demand in each market that clears is by definition equal to zero, in each case of market disequilibrium the structure of the constraints in the disequilibrium model implies the excess demand for money must be equal to zero also.

V. Summary of Conclusions

The purpose of this paper has been to clarify the role of money and Walras' law within the general theory of market disequilibrium. It has been demonstrated that the theory of choice underlying the disequilibrium model implies Patinkin's textbook demand for money and that this concept of money demand is well defined and unambiguous. It has also been demonstrated that Walras' law is irrelevant to disequilibrium analysis and cannot be held in the disequilibrium model by arbitrarily redefining what is meant by the demand for money. Such a redefinition (1) implies Walras' law only in a tautological sense, (2) does not allow us to reduce the number of equations in the disequilibrium model, and (3) leads to a concept of money demand that is empirically meaningless. It has been further demonstrated that the structure of constraints in the disequilibrium model implies that corresponding to each case of market disequilibrium there exists an aggregate constraint on the system as a whole, which states that the sum of the excess demands in those markets which clear plus the excess demand for money is equal to zero. As a result, (1) in the analysis of each case of market disequilibrium the number of equations can be reduced either by one of the market clearing equations, or by the excess demand for money equation, without affecting the properties of the model, and (2) the excess demand for money in the disequilibrium model is always equal to zero and can have no direct effect on the process of price formation.

Bibliography


