demand curve (which is obtained by vertical summation).

Although the net demand curve model is limited in application due mainly to its two person, non-strategic behavioral assumptions it does appear, nevertheless, to be suitable for extensions to multi-individual situations and strategic behavior. While leading to the usual conclusion that public goods will be underpro-
vided by private markets, the model offers a slightly different perspective by emphasizing the real income effects of joint consumption and the resulting impact on the demand for public goods.

A number of authors have recently considered the impact of price uncertainty on the behavior of the cooperative firm operating in a competitive market. One interesting result demonstrated by Paroush and Kahana is that under price uncertainty the cooperative firm will employ a greater amount of labor input than a cooperative firm facing a non-random price equal to the mean price in the uncer-
ainty case. This result is in contrast to the capitalist firm, where the firm facing price uncertainty employs a lesser amount of labor than the firm facing a non-random price.

In this paper this result is reexamined in a situation where the firm faces not only an uncertain future spot price for output, but also a current non-random futures price. It is demonstrated that under such an environment the cooperative firm will employ exactly the same amount of labor as a cooperative firm facing a non-random price equal to the current futures price.

Another related issue which will be expli-
citly considered in this paper is under what conditions the cooperative will hedge or specu-
late in the market. Specifically, it will be demonstrated that the "hedge versus specu-
late" issue will depend on the relationship between the futures price and the mean of the future spot price.

One other question to be considered in this paper is whether the existence of futures markets induces the cooperative firm to increase or decrease output. Specifically, it will be shown that the answer to this question is different for the cooperative firm and the capitalist firm.

In the certainty case, the objective function of the cooperative firm can be written as,

\[ \Pi = (P_f - \bar{W} - FC)/\bar{X}, \]

(1)

where \(\Pi\) is the profit per unit labor, \(X\) is the number of units of labor, \(W\) is the wage and FC is the fixed costs associated with the firm. \(P_f\) is the certain price equal to the current futures price in the uncertainty case and \(f\), which is a function of \(X\), represents the production technology of the firm with the assumption that \(f > 0\) and \(f^* < 0\).

The profit function of the cooperative firm facing both an uncertain future spot market

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See, for example, Paroush and Kahana (1980), Ramas frosthendron, et al. (1979) and Taub (1974). Irrespec-
tively, an incorrect version of Taub's paper was published by the FEJ. A correct version can be obtained from the author.

*Ref.

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and a certain current futures market can be written as:

$$\Pi = \frac{P(f - S) + (P_f S - WX - FC)(1 + r)}{X}$$

(2)

$P$ is the random future spot price, $S$ represents sales in the futures market, $r$ is the rate of interest, and $(f - S)$ represents sales in the future spot market. Sales in the current futures market along with wages and fixed costs are transacted today while sales in the future spot market occur one period hence. Thus dollars today are converted into dollars in the future by $(1 + r)$.

The cooperative firm maximizing $\Pi$, will hire $X$ so that,

$$\frac{d\Pi}{dX} = \frac{P_f X'(f - f) + FC}{X^3} - \frac{X^2}{X^3} = 0 - P_f X'(f - f) + FC.$$  

(3)

This result has already been demonstrated by many authors (Ward, 1958).

It is assumed that the cooperative firm facing both a futures market and future spot market maximizes expected utility of $\Pi$. Thus the problem is to choose $X$ and $S$ so that $EU[\Pi^2]$ is maximized. The first order conditions for a maximum are given by the following two equations:

$$EU^* = \frac{P_f X'(f - f) + FC}{X^3} = 0$$

$$EU^* = \frac{P_f (1 + r) - P}{X} = 0$$  

(4)

The first result of this paper is that the cooperative firm facing an uncertain future spot price and a certain futures price will choose $X$ as if it faces only a certain price equal to the futures price. This result can be shown by rewriting equations (4) as,

$$(X' - f + S)\left[\frac{P_f S - FC}{(1 + r)}\right] = EU^* [EU^* - P].$$

(5)

Substituting one equation into the other finds the condition

$$P_f X'(f - f) + FC = 0.$$  

(6)

This condition is exactly the same as for the cooperative firm facing a certain price equal to $P_f$.

The reason why $X$ is determined only by the certain price in the futures market can be seen by rewriting profit per worker as,

$$\Pi^* = \Pi^*(1 + r) + \frac{(f - S)(P - P_f(1 + r))}{X}.$$  

(7)

The first term on the right represents the non-random profits per worker assuming that the cooperative sells its total output in the futures market. The second term on the right represents random profits from sales (purchases) in the future spot market. If $f > S$ the cooperative hedges partially (completely) in the futures market, while, if $f < S$ or $S < 0$ the firm speculates in the future spot market. The cooperative's profit from participating in the spot market is based on the difference between the random spot price and the non-random futures price. Thus the firm's decision can be considered as if it occurred sequentially. First, it produces and sells all its output in the non-random futures market. This decision determines the optimal level of $X$. If the cooperative then wishes to hedge partially, it reprograms some output in the futures market to sell in the spot market. Similarly, if the cooperative speculates and $S < 0$, it repurchases more than its total output to sell in the spot market. If $f < S$, it purchases in the spot market to resell in the futures market.

This interpretation of the previous result

implies that the firm's decision on whether to hedge or speculate is independent of $X$ and, therefore, independent of whether the firm is a capitalist or cooperative firm. In fact, this result can be shown explicitly. Rewriting the second equation in (4) one gets,

$$EU^* = \frac{1}{X^2} \left[ \frac{P_f (1 + r) - P}{X} \right] + COV\left(U, P_f (1 + r) - P\right) = 0.$$  

(8)

If $P_f (1 + r) - EP$, the first term in (8) is zero and, therefore, the covariance term in (8) must also be zero. Now for $f - S$ it can be seen from (7) that $\Pi^*$ is independent of $P$ and, thus, the covariance term in (8) is zero (since $U$ is a function of $\Pi^*$). If $f > S$, $\Pi^*$ varies directly with $P$. Thus increases in $P$ increase $\Pi^*$ and decrease $U$. This implies that the covariance term in (8) is positive. Thus, if $P_f (1 + r) - EP$ then $f > S$. In a similar fashion, one can show that $P_f (1 + r) > EP$ implies $f < S$. Thus, the decision whether to hedge or speculate is seen to depend solely on the relation between $P_f (1 + r)$ and $EP$. This result has already been shown by Holtzhausen for the capitalist firm.

In a similar fashion, one can show that the effect of the degree of risk aversion or the riskiness of the spot price distribution on the hedging decision of the cooperative firm is exactly the same as for the capitalist firm. These results can be summarized as follows:

- If $f > S$ the firm will hedge more as it is more risk averse. If $f < S$ the firm will speculate less as it becomes more risk averse.

One issue where there exists a difference between the capitalist firm and the cooperative firm is whether the existence of futures markets causes production by the cooperative to be greater or less than it would be without the futures markets. Holtzhausen has shown that for the capitalist firm, the existence of futures markets causes a firm to increase output providing that the optimal hedge is positive. If the optimal hedge is negative, the existence of futures markets would cause output to fall. However, a similar result does not hold for the cooperative firm. To verify this, we compare the optimal output of a cooperative firm which is constrained at a zero hedge to the output of a firm which is free to choose both its optimal output and hedge. The latter case has already been considered in the first part of this paper.

To determine the optimal output of a cooperative firm constrained at a zero hedge, we maximize the expected utility of (2) subject to the constraint $S = 0$. The first order conditions for a maximum are,

$$EU^* = \left[ P_f X'(f - f) + (FC - P_f S)(1 + r) \right] = 0.$$  

(9)

Recognizing that $EU^* P = EU^* EP + COV(U, P)$, these conditions can be rewritten as,

$$[EP(1 + r)(X' - f) + FC] = \frac{1}{X^2} \left[ \frac{P_f (1 + r) - P}{X} \right] + COV(U, P_f (1 + r) - EP).$$  

(10)

Substituting (11) into (10) and recognizing,

$$S = 0.$$  

(11)

This statement refers to the decision on whether to hedge or speculate not the actual amounts produced and hedged.

See Holtzhausen (1979), pages 991-992.

\textsuperscript{3}Ibid., page 992.
that the sign of \( C O V(U', P) \) is equal to the sign of \(-U'(1 - \gamma)\), we then have,

\[
\text{Sign} \left( E P (1 + r)(X^{*} - f) + FC \right) = \text{Sign} \left(-U'(X^{*} - f)\right) < 0. \tag{12}
\]

(12) is less than zero since \( X^{*} - f < 0 \) and \( U' < 0 \) for a risk averse firm.

Consider now a firm which can choose both \( X \) and \( S \). The optimal amount of labor, \( X^{*} \), is determined by, (see (6))

\[
P_{e}(X^{*} - f) + FC = 0. \tag{6}
\]

Suppose the optimal hedge, \( S^{*} \), is \( S^{*} < f \). This implies that \( E P (1 + r) < P_{e} \). For such a firm we, therefore, have

\[
E P (1 + r)(X^{*} - f) + FC \leq P_{e}(X^{*} - f) + FC = 0. \tag{13}
\]

since \( X^{*} - f < 0 \). But since \( X^{*} - f \) is a decreasing function of \( X \), \( X^{*} < X_{lb} \) where \( X_{lb} \) is the optimal value of \( X \) when \( S \) is constrained to zero. Next, suppose that the optimal hedge is such that, \( S^{*} = 0 \). In this case \( E P (1 + r) > P_{e} \). However, when \( S^{*} = 0 \), \( X^{*} = X_{lb} \) since the first order condition in this case is exactly the same as when the firm is constrained to \( S = 0 \). Next, consider the case, \( S^{*} = 0 \). For \( S^{*} < 0 \),

\[
\left( E P (1 + r) - P_{e} \right)|_{S^{*} = 0} > \left( E P (1 + r) - P_{e} \right)|_{S^{*} = 0}. \tag{14}
\]

But this implies that \( X^{*} > X_{lb} \). Finally, consider the partial hedge case, where \( 0 < S^{*} < f \). In this situation \( E P (1 + r) - P_{e} \)|_{S^{*} = 0} < \( (E P (1 + r) - P_{e}) \)|_{S^{*} = 0}. This, however, implies that \( X^{*} < X_{lb} \).

These results indicate that for the cooperative firm the existence of futures markets causes the firm to decrease output, provided that the optimal hedge is positive. If the optimal hedge is negative, the existence of futures markets causes the firm to increase output. This result is the exact opposite of what Holthausen found for the capitalist firm.

The existence of futures markets causes the cooperative firm and the competitive firm to behave differently because of the difference in their reactions to price uncertainty. As Sandino has demonstrated, the capitalist firm, when faced with price uncertainty, will produce less than it will in the certainty case.3 The introduction of futures markets adds uncertainty to the uncertainty case, as long as the firm hedges positively. Thus, in this case the firm's output will be greater than its output when no futures markets exist. If the optimal hedge is negative, there is added uncertainty and the firm's output will be less than it would be if no futures markets exist. This is what Holthausen, and Fedor, et. al., have demonstrated. The cooperative firm's reaction to uncertainty, however, is to increase output. Therefore, if hedging is positive, the firm decreases output. If the optimal hedge is negative, the added uncertainty causes the firm to increase output.

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